Correct Equations for Revolving Door Kinetic Energy

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Herein is derived the correct equation for the rotational kinetic energy of a multi-wing, or multi-panel, revolving door. The following assumptions are made that characterize the door.

1. The rotating part of the door consists of a number n of identical flat rectangular panels ganged rigidly together and oriented radially, where n is typically 3 or 4.

2. The panels revolve around a common vertical axis coincident with one vertical edge of each panel.

3. The mass of each panel is distributed uniformly in the horizontal (radial) direction across the width of the panel. It is not necessary, and it is not assumed, that the panel mass is distributed uniformly in the direction parallel to the axis of rotation. That is, in the vertical direction.

4. The thickness of the panels is a small fraction of the width of the panels.

5. Other components of the door that may be in motion, such as a ceiling and the drive mechanism, are not considered.

The rotational kinetic energy of any mass distribution associated with rigid rotation of the mass distribution about a fixed axis is

$$E = \frac{1}{2} I \omega^2 , \tag{1}$$

where I is the moment of inertia of the mass distribution about the fixed axis of rotation, and ω is the angular velocity of the rotation in radians/second.[1]

As a consequence of the assumptions delineated on page 1, the moment of inertia of each door panel is equivalent to that of a uniform bar rotating about one end. That is, to

$$I = \frac{1}{3} M L^2 , \qquad (2)$$

where M is the mass of the bar and L is its length. Because of the equivalency just mentioned, M is here the mass of an individual door panel and L its width.

Equations (1) and (2) are the sole and entire basis for the derivation of the equation for the rotational kinetic energy of the door.

The rotational kinetic energy of the i-th panel of the revolving door is found by combining equations (1) and (2). That is

$$E_i = \frac{1}{6} M L^2 \omega^2 , \qquad (3)$$

where $i = 1, 2, 3... n$. Since the panels are assumed to be identical, the total kinetic energy of rotation of the door is simply n times that of an individual panel given by (3). That is, the total kinetic energy of rotation of the door is

$$E = \frac{1}{6} n M L^2 \omega^2 . \qquad (4)$$

To express the kinetic energy in units of pound-feet, the units used by the ANSI A156.27 national standard for automatic revolving doors[2], mass M must be expressed in "equivalent" pounds and the width L of an individual panel expressed in feet. Also, the angular velocity ω must be expressed as equivalent revolutions per second for compatibility with ANSI A156.27.

The pound is *not* a unit of mass, but a unit of force. But, with the understanding that weight in pounds is determined at sea level, the weight w in pounds that corresponds to

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[1] Specifically, the moment of inertia is the second spatial moment of the mass distribution about the axis of rotation.

the mass M (in slugs) of an individual door panel at sea level is given by Newton's first law, which relates force and mass, as

$$w = Mg\,, \quad \text{or} \quad M = \frac{w}{g}\,, \tag{5}$$

where g is the acceleration of gravity at sea level and has the approximate value 32.15 ft/sec^2.

Since one revolution is equal to 2π radians, and a minute contains 60 seconds,

$$\omega = \Omega\left(\frac{2\pi}{60}\right), \tag{6}$$

where Ω is the rotation rate of the door in revolutions per minute (rpm).

Substitution of (5) and (6) into (4) gives,

$$E = \frac{1}{6}\left(\frac{n\,w}{g}\right)\left(\frac{2\pi\,L}{60}\right)^2 \Omega^2\,. \tag{7}$$

And, substituting the numerical values for g and π gives,

$$E = \frac{n\,w\,L^2\Omega^2}{17590}\,, \tag{8}$$

where the total rotational kinetic energy E of the door is in units of pound-feet (lb-ft), n is the number of door panels, w is the weight of an individual door panel in pounds, L is the width of a door panel in feet, and Ω is the rotation rate of the door in revolutions per minute (rpm).[2]

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[2] The authors of ANSI A156.27-2003 indicate that the kinetic energy of the door is expressed in foot-pounds (ft-lb). However, a foot-pound is a unit of torque. Though dimensionally equivalent, the correct English unit of kinetic energy is the pound-foot (lb-ft).

Equation (8) can be used to calculate the rotation rate Ω of the door that results in the door carrying exactly 2.5 lb-ft of rotational kinetic energy by solving (8) for Ω with *E* set to 2.5 lb-ft. Namely,

$$\Omega_{2.5} = \frac{209.7}{L}\sqrt{\frac{1}{n\,w}}\,. \tag{9}$$

This can be brought into notational correspondence with the analogous equation in ANSI A156.27-2003 by recognizing that the diameter *D* of the door is twice the width *L* of an individual panel and that the total weight *W* of the door is *n w*. That is,

$$L = \frac{D}{2} \quad \text{and} \quad W = n\,w\,. \tag{10}$$

When relationships (10) are substituted into (9), the result is

$$\Omega_{2.5} = \frac{419}{D}\sqrt{\frac{1}{W}}\,. \tag{11}$$

This is the correct formula for the rotation rate in revolutions per minute of a multi-panel revolving door meeting the assumptions listed in page 1 and carrying exactly 2.5 lb-ft of rotational kinetic energy. For further comparison with ANSI A156.27-2003, the corresponding formula for a door carrying exactly 7.0 lb-ft of kinetic energy is

$$\Omega_{7.0} = \frac{702}{D}\sqrt{\frac{1}{W}}\,. \tag{12}$$

In equations (11) and (12), $\boldsymbol{\Omega}_{2.5}$ and $\boldsymbol{\Omega}_{7.0}$ are the rotation rates in rpm, *D* is the overall door diameter in feet, and *W* is the total combined weight of all door panels.

Note that the equations provided on page 16 of ANSI A156.27-2003 for these two rotation rates are ***incorrect***. They were derived using the naive, and quite incorrect, assumption that the entire mass of each revolving door panel is concentrated at the center of mass of the panel, with the result that the equations in ANSI A156.27-2003 give the rotation rates for doors in which the orientations of the individual door panels remain inertially fixed as the door rotates. That is, the individual panels continue to face always in the same direction, rather then changing their orientation as the door rotates. This is, of course, impossible if the individual panels maintain, as in a real revolving door, a common vertical edge coincident with the axis of rotation.

The result is that the equations of ANSI A156.27-2003, and Table 1 on page 15, which is derived from the incorrect equations, significantly ***underestimate*** the kinetic energy of the door at any given rotation rate. Conversely, they significantly ***overestimate*** the rotation rate of the door that results in either 2.5 lb-ft or 7.0 lb-ft of rotational kinetic energy.

Equation (8) may likewise be made to correspond to the notation of ANSI A156.27-2003 by substituting (10) to get

$$E = \frac{WD^2\,\Omega^2}{70360}\,. \tag{12}$$

Compare with the incorrect equation for *KE* (here *E*) under "KE at given RPM" at the lower right corner of page 16 of the ANSI A156.27-2003 standard, which reads

$$E = \frac{WD^2\,\Omega^2}{93091}\,. \tag{13}$$