Kinetic Energy Equations for 3-wing and 4-wing Revolving Doors with a Core

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Herein are derived the equations for the rotational kinetic energy of 3-wing and 4-wing revolving doors with a polygonal core. The following assumptions are made that characterize the door.

1. The rotating part of the door consists of a number *n* of identical flat rectangular radially directed panels each rigidly attached along one vertical edge to one vertex of an *n*-sided regular polygonal core.

2. The *n* sides of the regular polygonal core to which the radial panels are attached consist also of identical flat rectangular panels.

3. The core to which the panels are attached revolves around a vertical axis coincident with the geometric center of the polygonal core.

4. The mass of each panel is distributed uniformly in the horizontal direction across the width of the panel. This applies to the radial panels and the panels that comprise the core alike. It is not necessary, and it is not assumed, that the panel mass is distributed uniformly in the direction parallel to the axis of rotation. That is, in the vertical direction.

5. The thickness of the panels is a small fraction of the width of the panels.

6. Other components of the door that may be in motion, such as a ceiling and the drive mechanism, are not considered.

The figure below illustrates the dimensions and naming conventions employed in the derivations using a 3-wing door as an example. However, because the equations for two door configurations are to be derived – those for a 3-wing and those for a 4-wing door, the kinetic energy equations will first be derived for the general case of a door with *n* identical radial panels and an *n*-sided regular polygonal core. Then, *n* will be particularized to 3 and 4.

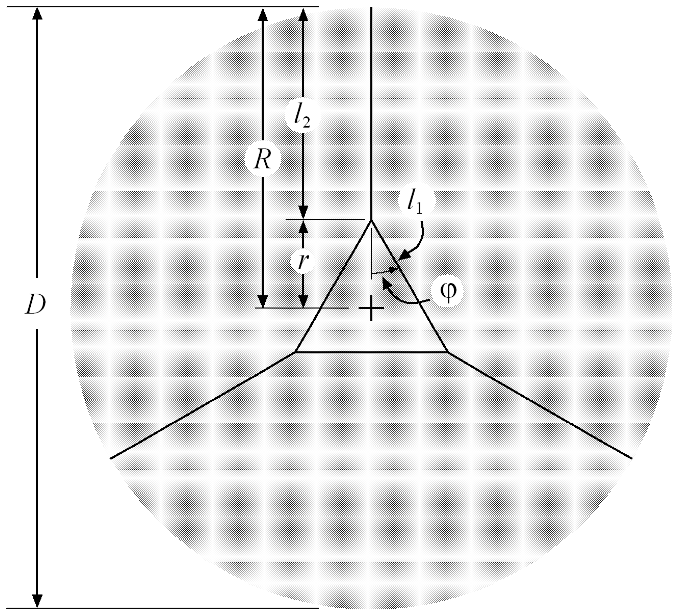

Figure 1. Dimensions and naming conventions for a revolving door with a regular polygonal core.

D represents the overall diameter of the door, *R* the radius of the door, *r* the radial distance from the geometric center of the door to a vertex of the polygonal core, l_1 the width of one side of the regular polygonal core, l_2 the width of each radial panel, and φ the internal half-angle subtended by a vertex of the regular polygonal core.

The derivation of the kinetic energy equations for the complete door will start with the derivation of the kinetic energy contributed by a basic unit consisting of a radial panel and one of the core panels to which it is attached. The complete door comprises *n* such units so that the kinetic energy of the complete door will be *n* times the result derived for the basic unit. The basic unit is illustrated in the figure below consisting of the radial panel l_2 and core panel l_1.

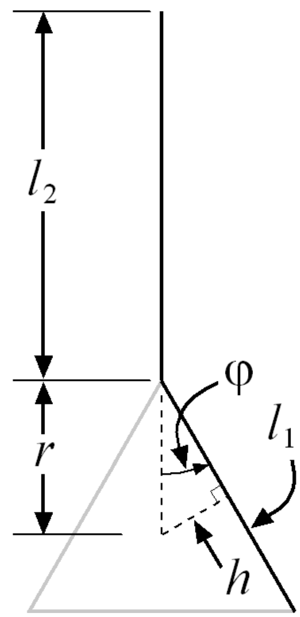

Figure 2.

h is the perpendicular distance from the geometric center of the regular polygonal core to the center of one side of the core.

The following relationships are clear from Figure 2.

$$h = r\sin\varphi \tag{1}$$

and

$$l_1 = 2r\cos\varphi\,. \tag{2}$$

It is also clear from Figure 1 that

$$R = \frac{D}{2} = l_2 + r\,, \quad \text{or} \quad r = \frac{D}{2} - l_2\,. \tag{3}$$

The interior angle subtended by the vertex of a regular plane polygon of n sides is

$$\pi\left(\frac{n-2}{n}\right).$$

Consequently,

$$\varphi = \frac{\pi}{2}\left(\frac{n-2}{n}\right). \tag{4}$$

The rotational kinetic energy of any mass distribution associated with rigid rotation of the mass distribution about a fixed axis is

$$E = \frac{1}{2} I\omega^2 , \tag{5}$$

where I is the moment of inertia of the mass distribution about the fixed axis of rotation, and ω is the angular velocity of the rotation in radians/second.[1]

As a consequence of the assumptions delineated on page 1, the moment of inertia of each door panel, radial and core, about its center-of-mass is equivalent to that of a uniform bar rotating about its center. That is, to

$$I = \frac{1}{12} M\, L^2 \tag{6}$$

where M is the mass of the bar and L is its length. Because of the equivalency just mentioned, M is here the mass of an individual door panel and L its width. Combining (5) and (6), the rotational kinetic energy of a panel of mass M and width L due to its rotation about its own center-of-mass is

$$E^{(cm)} = \frac{1}{24} M\, L^2 \omega^2 . \tag{7}$$

The kinetic energy due to the rotation of the center-of-mass of the panel about the axis of rotation of the door is

$$E^{(orbit)} = \frac{1}{2} M \rho^2 \omega^2 \tag{8}$$

where ρ is the radial distance of the center-of-mass of the panel from the axis of rotation of the door.

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[1] Specifically, the moment of inertia is the second spatial moment of the mass distribution about the axis of rotation.

The total kinetic energy of the panel due to its rigid rotation within the structure of the door is the sum of the two components given by (7) and (8).[2] Namely,

$$E^{(panel)} = E^{(cm)} + E^{(orbit)}. \tag{9}$$

Let the mass of a core panel be m_1 and that of a radial panel m_2. From (7), the kinetic energy associated with the rotation of a core panel about its own center-of-mass is

$$E_1^{(cm)} = \frac{1}{24} m_1 \omega^2 l_1^2 = \frac{1}{6} m_1 \omega^2 r^2 \cos^2\varphi = \frac{1}{6} m_1 \omega^2 r^2 \left(1 - \sin^2\varphi\right), \tag{10}$$

where (2) for l_1 has also been introduced. And, from (8), the kinetic energy associated with the rotation of the center-of-mass of a core panel about the axis of rotation of the door is

$$E_1^{(orbit)} = \frac{1}{2} m_1 \omega^2 h^2 = \frac{1}{2} m_1 \omega^2 r^2 \sin^2\varphi, \tag{11}$$

where (1) for h has been introduced. Consequently, the total kinetic energy of a core panel is, from (9), (10) and (11),

$$E_1^{(panel)} = \frac{1}{2} m_1 \omega^2 r^2 \left(\sin^2\varphi + \frac{1 - \sin^2\varphi}{3} \right) = \frac{1}{6} m_1 \omega^2 r^2 \left(1 + 2\sin^2\varphi\right). \tag{12}$$

And, with the substitution of (3) for r in (12),

$$E_1^{(panel)} = \frac{1}{6} m_1 \omega^2 \left(\frac{D}{2} - l_2 \right)^2 \left(1 + 2\sin^2\varphi\right). \tag{13}$$

In the same manner, the kinetic energy associated with the rotation of a radial panel about its own center-of-mass is from (7)

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[2] "Rigid" here means that, due to the fact that the panel is rigidly attached to the structure of the door, it is forced to rotate about its own center-of-mass as well as to rotate as a whole about the axis of rotation of the door. Equation (9) accounts for both of these rotational components that characterize the motion of the panel.

$$E_2^{(cm)} = \frac{1}{24} m_2 \, \omega^2 \, l_2^2 \, . \tag{14}$$

Since the mass distribution of the radial panel is uniform across the width of the panel, the center-of-mass of the radial panel is located at a distance $l_2/2$ from either edge (side) of the panel. Referring to either Figure 1 or 2, it is clear then that the radial distance of the center-of-mass of a radial panel from the axis of rotation of the door is

$$\rho = r + \frac{l_2}{2} = \left(\frac{D}{2} - l_2 \right) + l_2 = \frac{D - l_2}{2} \, , \tag{15}$$

where (3) for r has been introduced.

The kinetic energy associated with the rotation of the center-of-mass of a radial panel about the axis of rotation of the door is found from (8) and (15) to be

$$E_2^{(orbit)} = \frac{1}{2} m_2 \, \omega^2 \rho^2 = \frac{1}{8} m_2 \, \omega^2 \left(D - l_2 \right)^2 . \tag{16}$$

The kinetic energy of the combination of a core panel and a radial panel, the basic unit considered here, is

$$E^{(unit)} = E_1^{(cm)} + E_1^{(orbit)} + E_2^{(cm)} + E_2^{(orbit)} = E_1^{(panel)} + E_2^{(cm)} + E_2^{(orbit)} \, ,$$

which, from (13), (14) and (16), is found to be

$$E^{(unit)} = \frac{1}{6} m_1 \omega^2 \left(\frac{D}{2} - l_2 \right)^2 \left(1 + 2\sin^2 \varphi \right) + \frac{1}{8} m_2 \omega^2 \left[\left(D - l_2 \right)^2 + \frac{l_2^2}{3} \right] . \tag{17}$$

Clearing the fractions from the factors in parentheses and brackets and taking ω to the left gives

$$E^{(unit)} = \frac{\omega^2}{24} \left\{ m_1 \left(D - 2l_2 \right)^2 \left(1 + 2\sin^2 \varphi \right) + m_2 \left[3 \left(D - l_2 \right)^2 + l_2^2 \right] \right\} . \tag{18}$$

And expanding the factors involving D gives

$$E^{(unit)} = \frac{\omega^2}{24}\left\{m_1\left(D^2 - 4Dl_2 + 4l_2^2\right)\left(1 + 2\sin^2\varphi\right) + m_2\left(3D^2 - 6Dl_2 + 4l_2^2\right)\right\} \quad (19)$$

Result (19) can now be particularized for a 3-wing or 4-wing door by setting *n* equal to 3 or 4, respectively. With reference to (4),

$$\sin\varphi = \sin\frac{\pi}{6} = \frac{1}{2}, \qquad (n = 3) \quad (20a)$$

$$\sin\varphi = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}, \qquad (n = 4) \quad (20b)$$

so that the trigonometric factor in the first term of (19) becomes

$$1 + 2\sin^2\varphi = 1 + 2\left(\frac{1}{2}\right)^2 = \frac{3}{2} \qquad (n = 3) \quad (21a)$$

$$1 + 2\sin^2\varphi = 1 + 2\left(\frac{1}{\sqrt{2}}\right)^2 = 2 \qquad (n = 4) \quad (21b)$$

Kinetic Energy Equations for a 3-wing Door with a Polygonal Core

A 3-wing door with a polygonal core is comprised of *n* = 3 units of the type depicted in Figure 2. Thus,

$$E_{3-wing} = nE^{(unit)} = 3E^{(unit)}, \quad (22)$$

where (19) for $E^{(unit)}$ is to be evaluated for the case *n* = 3 using (21a). Thus, (19), (21a) and (22) yield

$$E_{3-wing} = \frac{\omega^2}{8}\left[\frac{3}{2}m_1\left(D^2 - 4Dl_2 + 4l_2^2\right) + m_2\left(3D^2 - 6Dl_2 + 4l_2^2\right)\right]. \quad (23)$$

Grouping by descending powers of *D* yields

$$E_{3-wing} = \frac{\omega^2}{8}\left[D^2\left(\frac{3}{2}m_1 + 3m_2\right) - 6Dl_2\left(m_1 + m_2\right) + l_2^2\left(6m_1 + 4m_2\right)\right]. \qquad (24)$$

And taking out a factor of 6 gives

$$E_{3-wing} = \frac{3}{4}\omega^2\left[\left(\frac{D}{2}\right)^2\left(m_1 + 2m_2\right) - Dl_2\left(m_1 + m_2\right) + l_2^2\left(m_1 + \frac{2}{3}m_2\right)\right]. \qquad (25)$$

To express this kinetic energy in units of pound-feet, the units used by the ANSI A156.27 national standard for automatic revolving doors, the masses m_1 and m_2 must be expressed in "equivalent" pounds and the width of the radial panel l_2 and the overall door diameter D expressed in feet. Also, the angular velocity ω must be expressed as equivalent revolutions per second for compatibility with ANSI A156.27.

The pound is *not* a unit of mass, but a unit of force. But, with the understanding that weight in pounds is determined at sea level, the weight w in pounds that corresponds to the mass m (in slugs) of an individual door panel at sea level is given by Newton's first law, which relates force and mass, as

$$w = mg\,, \quad \text{or} \quad m = \frac{w}{g}\,, \qquad (26)$$

where g is the acceleration of gravity at sea level and has the approximate value 32.15 ft/sec^2.

Since one revolution is equal to 2π radians, and a minute contains 60 seconds,

$$\omega = \left(\frac{2\pi}{60}\right)\Omega = \left(\frac{\pi}{30}\right)\Omega\,, \qquad (27)$$

where Ω is the rotation rate of the door in revolutions per minute (rpm).

Substituting (26) and (27) into (25) and using the numerical values for g and π gives,

$$E_{3-wing} = \frac{1}{3909}\left[\left(\frac{D}{2}\right)^2\left(w_1 + 2w_2\right) - Dl_2\left(w_1 + w_2\right) + l_2^2\left(w_1 + \frac{2}{3}w_2\right)\right]\Omega^2\,, \qquad (28)$$

where the total rotational kinetic energy E_{3-wing} of the door is in units of pound-feet (lb-ft), w_1 and w_2 are the weights of a core panel and a radial panel, respectively, in pounds, l_2 is the width of a radial panel and D the overall door diameter both in feet, and Ω is the rotation rate of the door in revolutions per minute (rpm).[3]

Equation (28) can be used to calculate the rotation rate Ω of the door that results in the door carrying exactly 2.5 lb-ft of rotational kinetic energy by solving (28) for Ω with E_{3-wing} set to 2.5 lb-ft. The result is,

$$\Omega_{2.5} = \frac{98.9}{\sqrt{\left(\frac{D}{2}\right)^2 (w_1 + 2w_2) - Dl_2(w_1 + w_2) + l_2^2\left(w_1 + \frac{2}{3}w_2\right)}} . \tag{29}$$

Equation (28) may likewise be used to determine the rotation rate Ω of the door that results in the door carrying exactly 7.0 lb-ft of rotational kinetic energy by solving (28) for Ω with E_{3-wing} set to 7.0 lb-ft. The result is,

$$\Omega_{7.0} = \frac{165}{\sqrt{\left(\frac{D}{2}\right)^2 (w_1 + 2w_2) - Dl_2(w_1 + w_2) + l_2^2\left(w_1 + \frac{2}{3}w_2\right)}} . \tag{30}$$

Kinetic Energy Equations for a 4-wing Door with a Polygonal Core

A 4-wing door with a polygonal core is comprised of n = 4 units of the type depicted in Figure 2. Thus,

$$E_{4-wing} = nE^{(unit)} = 4E^{(unit)} , \tag{31}$$

where (19) for $E^{(unit)}$ is now to be evaluated for the case n = 4 using (21b). Thus, (19), (21b) and (31) yield

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[3] The authors of ANSI A156.27-2003 indicate that the kinetic energy of the door is expressed in foot-pounds (ft-lb). However, a foot-pound is a unit of torque. Though dimensionally equivalent, the correct unit of kinetic energy is the pound-foot (lb-ft).

$$E_{4-wing} = \frac{\omega^2}{6}\left[2m_1\left(D^2 - 4Dl_2 + 4l_2^2\right) + m_2\left(3D^2 - 6Dl_2 + 4l_2^2\right)\right]. \quad (32)$$

Grouping by descending powers of D yields

$$E_{4-wing} = \frac{\omega^2}{6}\left[D^2\left(2m_1 + 3m_2\right) - 2Dl_2\left(4m_1 + 3m_2\right) + 4l_2^2\left(2m_1 + m_2\right)\right]. \quad (33)$$

And taking out a factor of 2 gives

$$E_{4-wing} = \frac{\omega^2}{3}\left[\frac{D^2}{2}\left(2m_1 + 3m_2\right) - Dl_2\left(4m_1 + 3m_2\right) + 2l_2^2\left(2m_1 + m_2\right)\right]. \quad (34)$$

Substituting (26) and (27) into (34) and using the numerical values for g and π gives,

$$E_{4-wing} = \frac{1}{8795}\left[\frac{D^2}{2}\left(2w_1 + 3w_2\right) - Dl_2\left(4w_1 + 3w_2\right) + 2l_2^2\left(2w_1 + w_2\right)\right]\Omega^2. \quad (35)$$

where the total rotational kinetic energy E_{4-wing} of the door is in units of pound-feet (lb-ft), w_1 and w_2 are the weights of a core panel and a radial panel, respectively, in pounds, l_2 is the width of a radial panel and D the overall door diameter both in feet, and Ω is the rotation rate of the door in revolutions per minute (rpm).

Equation (35) can be used to calculate the rotation rate Ω of the door that results in the door carrying exactly 2.5 lb-ft of rotational kinetic energy by solving (35) for Ω with E_{4-wing} set to 2.5 lb-ft. The result is,

$$\Omega_{2.5} = \frac{148}{\sqrt{\frac{D^2}{2}\left(2w_1 + 3w_2\right) - Dl_2\left(4w_1 + 3w_2\right) + 2l_2^2\left(2w_1 + w_2\right)}}. \quad (36)$$

Equation (35) may likewise be used to determine the rotation rate Ω of the door that results in the door carrying exactly 7.0 lb-ft of rotational kinetic energy by solving (35) for Ω with E_{4-wing} set to 7.0 lb-ft. The result is,

$$\Omega_{7.0} = \frac{248}{\sqrt{\frac{D^2}{2}\left(2w_1 + 3w_2\right) - Dl_2\left(4w_1 + 3w_2\right) + 2\,l_2^2\left(2w_1 + w_2\right)}}\ . \tag{37}$$